Exam #2 · Thursday, October 9, 2008

MATH 124 · Calculus I · Section 26 · Fall 2008 Name _____

Problem 1. Let $f(x) = x^{2x}$. Numerically approximate $f'(1)$ using difference quotients. Use at least three successively smaller values of h.

Problem 2. The period T (in seconds) of a pendulum depends on the length ℓ (in centimeters) of the pendulum, as follows:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where g is the acceleration due to gravity (a positive constant).

Part (a). What are the units of $dT/d\ell$?

Part (b). Find the formula for $dT/d\ell$.

Problem 3. The horizontal displacement H (in millimeters) of a weighted spring from its equilibrium position, at time t milliseconds from the start of an experiment, is

$$H(t) = 2.3 + e^{-2t}\cos(t).$$

Part (a). What are the units of $H''(t)$?

Part (b). Find $H''(t)$.

Problem 4. The reaction time T (in minutes) of a chemical experiment is a function of the amount m (in grams) of catalyst added to a solution. That is, $T = f(m)$.

Part (a). Describe the significance of $f(200)$, including units in your answer.

Part (b). Describe the significance of $f^{-1}(6)$, including units in your answer.

Part (c). Suppose you discover the following: As you repeat the experiment, with one more gram of catalyst each time, you find that the reaction time decreases. However, each additional gram has less and less effect. What is sign of $f''(m)$? Please justify your answer. (Sketch a graph if you like. If you do so, please label your axes and explain verbally what you are doing.)

Problem 5. Let $f(x) = \frac{1}{x^2+1}$.

Part (a). Find $f'(x)$.

Part (b). Find an equation for the tangent line to $f(x)$ at $x = 2$.

Problem 6. Some values of $V(t)$ and $H(t)$ and their first derivatives are given by the following table.

t	$V(t)$	$H(t)$	$V'(t)$	$H'(t)$
6	8	-7	0.5	-0.2

Part (a). Let $E(t) = V(t) + H(t)$. Find $E'(6)$.

Part (b). Let $E(t) = V(t)H(t)$. Find $E'(6)$.

Part (c). Let $E(t) = \frac{V(t)}{H(t)}$. Find $E'(6)$.

Problem 7. Let $g(x) = xe^{-ax}$. Assume a is a positive constant. Over what interval(s) is $g(x)$ increasing? (You will need to solve an inequality.)

Problem 8. Let $C(\theta) = \frac{\cos(\theta)-1}{\theta}$.

Part (a). Does $C'(1)$ exist? If so, find its value exactly. If not, explain why not.

Part (b). Does $C'(0)$ exist? If so, find its value exactly. If not, explain why not.