Exam #2 Study Guide · MATH 111 · Section 7 · Fall 2006

Disclaimers about the study guide:

- Exam 2 covers chapters 4, 5, and 6. While all *topics* on the exam will be taken from this study guide, the specific *questions* on the exam will not be identical to the ones you see here.
- In addition to consulting this guide, please review all homework problems for chapters 4-6.
- This study guide is longer than the exam will be.

Notes about the exam:

- When I ask you for the *exact value* of something, do the necessary algebra and obtain an answer using integers and radicals. Decimal approximations obtained using your calculator will receive zero credit on problems which ask for an exact value. For example, the exact value of $\sin(\pi/4)$ is $\sqrt{2}/2$, not 0.707.
- When I ask you to *verify* a trignometric identity, I want you to begin with either the left-hand side or the right-hand side (whichever turns out to be easier, although you can safely start with the left-hand side for problems I write), then use algebra and trig identities to turn it into the right-hand side.
- When I ask you to *derive* a trignometric identity, I want you to begin with something else (which I will specify), then use algebra and trig identities to turn it into the desired expression. Simply writing down the desired expression will earn no credit.
- None of the problems on this study guide, and none of the problems on exam 2, require a calculator. You will be free to use your calculator on the exam, but please be aware that if you find yourself using your calculator for problems on this study guide, you are probably misunderstanding the questions or not reinforcing the correct skills.
- I will not list out any of the below identities for you on the exam. You should know them.

Topics:

- Chapter 4: Graphs of circular functions; translations of graphs.
- Chapter 5: Trigonometric identities
- Chapter 6: Inverse functions; solving trigonometric equations.

Identities you need to memorize:

- Quotient identities:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$
$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

- Reciprocal identities:

$$\csc(x) = \frac{1}{\sin(x)}$$
$$\sec(x) = \frac{1}{\cos(x)}$$
$$\cot(x) = \frac{1}{\tan(x)}$$

- Cofunction identities:

$$\begin{aligned} \sin(x) &= \cos(\pi/2 - x) \\ \cos(x) &= \sin(\pi/2 - x) \end{aligned}$$

- Negative-angle identities:

$$\begin{aligned} \sin(-x) &= -\sin(x), \\ \cos(-x) &= \cos(x), \end{aligned}$$

- Pythagorean identity:

$$\cos^2(x) + \sin^2(x) = 1.$$

- Sum and difference formulas for sine and cosine:

$$\begin{aligned} \sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y) \\ \cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y) \end{aligned}$$

- Double-angle identities:

$$\begin{aligned} \sin(2x) &= 2\sin(x)\cos(x) \\ \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 1 - 2\sin^2(x) \\ &= 2\cos^2(x) - 1. \end{aligned}$$

- You should be able to combine these identities. For example,

$$\tan(-x) = -\tan(x)$$

follows from the quotient identity for tangent along with the negative-angle identities for sine and cosine.

- You should be able to derive the four double-angle identities by using the sum identities for sine and cosine and the Pythagorean identity, as follows:

$$\begin{aligned} \sin(2x) &= \sin(x + x) \\ &= \sin(x)\cos(x) + \cos(x)\sin(x) \quad \text{(Sum formula for sine)} \\ &= 2\sin(x)\cos(x) \quad \text{(Simplify)} \\ \cos(2x) &= \cos(x + x) \\ &= \cos(x)\cos(x) - \sin(x)\sin(x) \quad \text{(Sum formula for cosine)} \\ &= \cos^2(x) - \sin^2(x) \quad \text{(Simplify).} \end{aligned}$$

Now recall the Pythagorean identity:

$$\sin^2(x) + \cos^2(x) = 1$$

and so we can write

$$\sin^2(x) = 1 - \cos^2(x)$$

or

$$\cos^2(x) = 1 - \sin^2(x).$$

We can plug these into the double-angle formula for cosine to obtain:

$$\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x)\\ &= \cos^2(x) - (1 - \cos^2(x))\\ &= 2\cos^2(x) - 1\end{aligned}$$

and similarly:

$$\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x)\\ &= (1 - \sin^2(x)) - \sin^2(x)\\ &= 1 - 2\sin^2(x).\end{aligned}$$

Questions:

(1) Given graphs of $\sin(x)$, $\sin(2x)$, $2\sin(x)$, $\sin(x/2)$, and $\sin(x)/2$, be able to specify which graph is which.

(2) Given the function $y = -4\sin(\frac{x}{2} + 1)$, find the amplitude and phase shift.

(3) Given the function $y = \frac{1}{2}\cos(\frac{3\pi x}{2} + \frac{\pi}{4})$, find the average value and period.

(4) If $\tan(x) = 4.1$, what is $\tan(-x)$?

(5) If $\cos(\theta) = 3/4$ and θ is in quadrant I, what is $\sin(\theta)$?

(6) If $\cos(\theta) = 3/4$ and θ is in quadrant IV, what is $\sin(\theta)$?

(7) Writing question: If $\cos(\theta) = 3/4$, why can't θ be in quadrant III?

(8) Given a cosine function $y = f(x)$ with average value 8, period 12, amplitude 3, and maximum value 4, find $f(10)$. (Hint: First graph the function. Then you should be able to easily read off the answer.)

(9) Write an equation for a sine function which passes through all the points in the following data table:

x	0	2	4	6	8	10
y	-6	-4	-2	-4	-6	-4

(10) Writing question: How many solutions for $\sin(x) = 0.6$ are there in $0 \le x < 2\pi$, and why?

(11) Writing question: How many solutions for $\sin(x) = 0.0$ are there in $0 \le x < 2\pi$, and why?

(12) Writing question: How many solutions for $\sin(x) = -1.0$ are there in $0 \le x < 2\pi$, and why?

(13) Writing question: How many solutions for $\sin(x) = -1.2$ are there in $0 \le x < 2\pi$, and why?

(14) Find the exact value of $\cos(7\pi/6)$.

(15) Find the exact value of $\csc(5\pi/4)$.

(16) Find the exact value of $\tan(-\pi/3)$.

(17) Find the exact value of $\cos(75°)$.

(18) Find six exact values of x such that $\sin(x) = 1$.

(19) Find six exact values of x such that $\sin(x) = 1/2$.

(20) Find the exact value of $\sin(\cos^{-1}(-1/3))$.

(21) Find the exact value of $\cos(\arctan(5/3))$.

(22) Find an algebraic expression for $\cos(\arcsin(u))$.

(23) Find the exact value of $\arccos(\cos(-60°))$.

(24) Verify the following trigonometric identities:

- $\sin^2(x)\sec^2(x) + \sin^2(x)\csc^2(x) = \sec^2(x)$.
- $\tan(x)\sin(x) + \cos(x) = \sec(x)$.
- $(\sin(x) + \cos(x))^2 = \sin(2x) + 1$.

(25) Find all solutions for $0 \leq x < 2\pi$ (or, if you prefer, do it in degrees for $0° \leq x < 360°$):

- $2\sin(x) + 3 = 4$.
- $2\cos^2(2x) = 1 - \cos(2x)$.
- $2\sin^2(x) - \sin(x) - 1 = 0$.
- $\sin^2(x) - 3\sin(x) = -2$.

(26) Writing question: Why is it that $\cos(\cos^{-1}(x)) = x$ always, while $\cos^{-1}(\cos(3\pi)) = \pi$?

(27) What are the domain and range of $\sin^{-1}$?

(28) What are the domain and range of $\cos^{-1}$?

(29) What are the domain and range of $\tan^{-1}$?

(30) What are $\sin^{-1}(-2)$, $\sin^{-1}(-1)$, $\sin^{-1}(0)$, $\sin^{-1}(1)$, and $\sin^{-1}(2)$? Is there any x such that $\sin^{-1}(x) = -\pi$, $-\pi/2$, 0, $\pi/2$, and/or π?

(31) What are $\cos^{-1}(-2)$, $\cos^{-1}(-1)$, $\cos^{-1}(0)$, $\cos^{-1}(1)$, and $\cos^{-1}(2)$? Is there any x such that $\cos^{-1}(x) = -\pi$, $-\pi/2$, 0, $\pi/2$, and/or π?

(32) What are $\tan^{-1}(-1)$, $\tan^{-1}(0)$, and $\tan^{-1}(1)$? Are $\tan^{-1}(-2)$ and $\tan^{-1}(2)$ defined? Is there any x such that $\tan^{-1}(x) = -\pi$, $-\pi/2$, 0, $\pi/2$, and/or π?

(33) Writing question: Why is $\arcsin(3)$ undefined?

(34) Solve for x:

$$y = 4\tan(3x).$$

(35) Solve for x:

$$y = 7\sin(2x) - 4.$$

(36) Solve for x:

$$3\sin^{-1}\left(\frac{x-\pi}{3}\right) = \frac{3\pi}{2}.$$