Lattice doubling for three-dimensional quantum networks

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Abstract

We extend a two-dimensional lattice-doubling result to three dimensions, in the context of percolation through quantum networks. This is a write-up for the final component of my independent-study course under Janek Wehr in the spring of 2008.

Contents

1 Review of the two-dimensional case

Our goal is to extend the results of section VI.C of [**PCALW**] from two dimensions to three. We begin by summarizing the two-dimensional situation.

xxx

Let

$$\pi = P[A \in C_\infty \cup A' \in C_\infty] = P[B \in C_\infty \cup B' \in C_\infty].$$

We want an upper bound on π^2. xxx compare to other.

xxx θ.

Using the inclusion-exclusion principle, we have

$$P[A \in C_\infty \cup A' \in C_\infty] = P[A \in C_\infty] + P[A' \in C_\infty] - P[A' \in C_\infty \cap A' \in C_\infty].$$

The first two terms are both θ. Using the transitivity of the clustering relation we may rewrite the last term as well. One obtains

$$P[A \in C_\infty \cup A' \in C_\infty] = 2\theta - P[A \in C_\infty \cap A \multimap\!\circ\, A'].$$

We now desire a lower bound on the last term. Using the FKG inequality [**Gri**] [xxx quack about increasing events],

$$P[A \in C_\infty \cap A \multimap\!\circ\, A'] \geq P[A \in C_\infty] P[A \multimap\!\circ\, A'].$$

Now, $P[A \in C_\infty]$ is simply θ; write

$$\tau = P[A \multimap\!\circ\, A'].$$

Then

$$P[A \in C_\infty \cap A \multimap\!\circ\, A'] \geq \theta\tau.$$

For our upper bound on π we now have

$$\pi \leq \theta(2 - \tau).$$

2 The three-dimensional case

2.1 Lattice doubling

xxx picture here

2.2 Upper bounds

2.3 Monte Carlo estimation of connectivity functions

2.4 Curve fitting

3 Conclusion

References

[**Gri**] Grimmett, G. *Percolation* (2nd ed.). Springer, 1999.

[**Mer**] Mermin, X. *Lecture Notes on Quantum Computation*. `http://people.ccmr.cornell.edu/~mermin/qcomp/CS483.html`.

[**NC**] Nielsen, M.A. and Chuang, I.L. *Quantum Computation and Quantum Information*. Cambridge, 2001.

[**PCALW**] Perseguers, S., Cirac, I., Acín, A., Lewenstein, M., and Wehr, J.. *Entanglement Distribution in Pure-State Quantum Networks*. `arxiv.org:0708.1025v2`.

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