Notes for functional analysis

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Abstract

This is a crib sheet for functional analysis. It is under construction.

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1 Introduction

Note that this is nowhere near being thorough. It's a handy reference for those topics which I care about. In particular, focusing on Hilbert spaces rather than Banach spaces.

bib: simon, lax, akhiezer and glazman, kolmogorov and fumin.

2 Finite-dimensional case

matrices: symmetric, hermitian; skew-symmetric, skew-hermitian; orthogonal, unitary. geometric reasons we care about these.

$$\begin{split} \langle Ax,Ay\rangle &= \langle x,y\rangle; \ \langle Ax,y\rangle = \langle x,Ay\rangle.\\ U^t &= U^{-1}; \ A^t = A. \end{split}$$

examples of matrices with various spectra.

wiki: compact operator on hilbert space.

3 Infinite-dimensional case

are there interesting examples of non-separable hilbert spaces?

boundedness of operators.

compact operator — multiple definitions. examples.

compact operator vs. continuous spectrum? examples.

examples of operators with empty, finite, countable, uncountable spectrum.

wik article on Compact operator on Hilbert space

examples of differences between symmetric and self-adjoint operators.

properties of d/dx and d^2/dx^2 .

examples of difference between eigenvalues and spectrum.

for all examples: are they physical, or pathological?

Some bases for $L^2(\mathbb{R})$, $L^2(S^1)$, $L^2(S^2)$, $L^2(S^3)$. What other manifolds / Lie groups?

References

[Simon] B. Simon Representations of Finite and Compact Groups. American Mathematical Society, 1996.

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